

NONLINEAR PROBLEM HAVING NATURAL GROWTH TERM AND MEASURE DATA

IBRAHIME KONATÉ¹, IBRANGO IDRISSA² AND STANISLAS OUARO³

ABSTRACT. The aim of this paper is to study the existence of solutions of multi-valued nonlinear elliptic problems involving the natural growth term, measure data and the general $p(\cdot)$ -Leray-Lions type operator. Using a decomposition of Radon diffuse measure due to Nyanquini et al. (see [26]) and approximation method, we construct an approximate problem and solve it by the technic of maximal monotone operators in Banach spaces. We prove that the approximate solutions converge to the solution of the studied problem.

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2020 *Mathematics Subject Classification*. 35J15, 35J20, 35J60, 35J67.

Key words and phrases. Generalized Lebesgue-Sobolev spaces, Leray-Lions operator, truncations, maximal monotone graph.

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Received 25 October 2023

¹ DÉPARTEMENT DE MATHÉMATIQUES, UNIVERSITÉ THOMAS SANKARA, 12 BP 417 OUGADOUGOU 12, BURKINA FASO, LABORATOIRE DE SCIENCES ET TECHNOLOGIE (LAST)

² DÉPARTEMENT DE MATHÉMATIQUES, LABORATOIRE DE MATHÉMATIQUES, D'INFORMATIQUE ET APPLICATIONS (LAMIA), UNIVERSITÉ NAZI BONI, 01 BP 1091 BOBO 01, BOBO DILOUSSO, BURKINA FASO

³ DÉPARTEMENT DE MATHÉMATIQUES, LABORATOIRE DE MATHÉMATIQUES ET D'INFORMATIQUE (LAMI), UNIVERSITÉ JOSEPH KI-ZERBO, 03 BP 7021 OUAGA 03, OUGADOUGOU, BURKINA FASO
Email address: ¹ ibrakonat@yahoo.fr, ² ibrango2006@yahoo.fr, ³ ouaro@yahoo.fr